Series : Z6YWX **SET~1**

प्रश्न-पत्र कोड
Q.P. Code **65/6/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे* *अधिकतम अंक : **80***

*Time allowed : **3** hours* *Maximum Marks : **80***

नोट		NOTE	
(I)	कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।	(I)	Please check that this question paper contains **23** printed pages.
(II)	प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।	(II)	Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
(III)	कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।	(III)	Please check that this question paper contains **38** questions.
(IV)	**कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।**	(IV)	**Please write down the Serial Number of the question in the answer-book at the given place before attempting it.**
(V)	इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।	(V)	15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. #

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. Let both AB' and $B'A$ be defined for matrices A and B. If order of A is $n \times m$, then the order of B is :

(A) $n \times n$ (B) $n \times m$

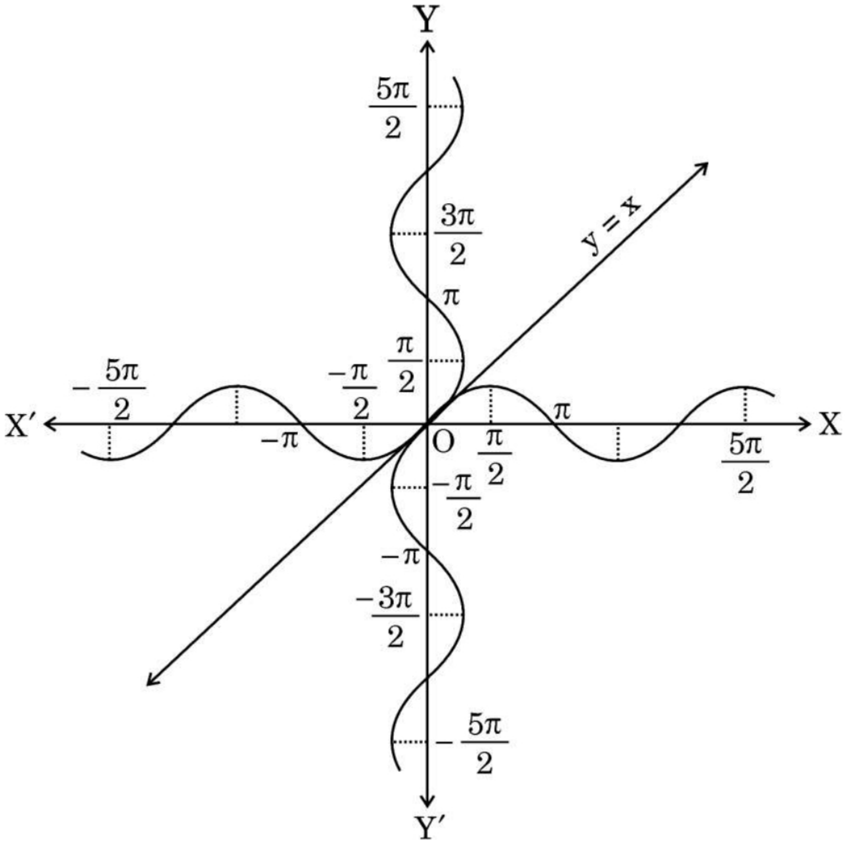
(C) $m \times m$ (D) $m \times n$

2. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A is a/an :

(A) scalar matrix (B) identity matrix

(C) symmetric matrix (D) skew-symmetric matrix

3. The following graph is a combination of :



(A) $y = \sin^{-1} x$ and $y = \cos^{-1} x$

(B) $y = \cos^{-1} x$ and $y = \cos x$

(C) $y = \sin^{-1} x$ and $y = \sin x$

(D) $y = \cos^{-1} x$ and $y = \sin x$

4. Sum of two skew-symmetric matrices of same order is always a/an :

(A) skew-symmetric matrix

(B) symmetric matrix

(C) null matrix

(D) identity matrix

5. $\left[\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ is equal to :

(A) $\frac{11\pi}{12}$ (B) $\frac{5\pi}{12}$

(C) $-\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$

6. If $f(x) = \begin{cases} \dfrac{\log(1+ax)+\log(1-bx)}{x}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$

is continuous at x = 0, then the value of k is :

(A) a (B) a + b

(C) a – b (D) b

7. If $\tan^{-1}(x^2 - y^2) = a$, where 'a' is a constant, then $\dfrac{dy}{dx}$ is :

(A) $\dfrac{x}{y}$ (B) $-\dfrac{x}{y}$

(C) $\dfrac{a}{x}$ (D) $\dfrac{a}{y}$

8. If y = a cos (log x) + b sin (log x), then $x^2y_2 + xy_1$ is :

(A) cot (log x) (B) y

(C) – y (D) tan (log x)

9. Let f(x) = |x|, x ∈ R. Then, which of the following statements is ***incorrect*** ?

(A) f has a minimum value at x = 0.

(B) f has no maximum value in R.

(C) f is continuous at x = 0.

(D) f is differentiable at x = 0.

10. Let $f'(x) = 3(x^2 + 2x) - \dfrac{4}{x^3} + 5$, f(1) = 0. Then, f(x) is :

(A) $x^3 + 3x^2 + \dfrac{2}{x^2} + 5x + 11$ (B) $x^3 + 3x^2 + \dfrac{2}{x^2} + 5x - 11$

(C) $x^3 + 3x^2 - \dfrac{2}{x^2} + 5x - 11$ (D) $x^3 - 3x^2 - \dfrac{2}{x^2} + 5x - 11$

11. $\displaystyle\int \frac{x+5}{(x+6)^2} e^x \, dx$ is equal to :

(A) log (x + 6) + C (B) $e^x + C$

(C) $\dfrac{e^x}{x+6} + C$ (D) $\dfrac{-1}{(x+6)^2} + C$

12. The order and degree of the following differential equation are, respectively :

$$-\frac{d^4y}{dx^4} + 2e^{dy/dx} + y^2 = 0$$

(A) $-4, 1$ (B) 4, not defined

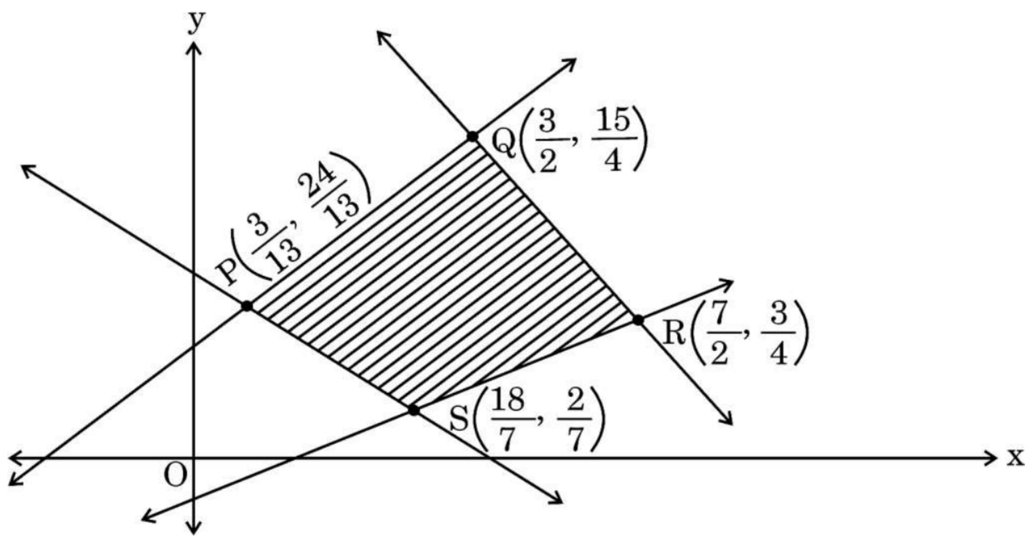
(C) 1, 1 (D) 4, 1

13. The solution for the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is :

(A) $3e^{4y} + 4e^{-3x} + C = 0$ (B) $e^{3x+4y} + C = 0$

(C) $3e^{-3y} + 4e^{4x} + 12C = 0$ (D) $3e^{-4y} + 4e^{3x} + 12C = 0$

14. For a Linear Programming Problem (LPP), the given objective function is $Z = x + 2y$. The feasible region PQRS determined by the set of constraints is shown as a shaded region in the graph.



(Note : The figure is not to scale)

$$P \equiv \left(\frac{3}{13}, \frac{24}{13}\right), Q \equiv \left(\frac{3}{2}, \frac{15}{4}\right), R \equiv \left(\frac{7}{2}, \frac{3}{4}\right), S \equiv \left(\frac{18}{7}, \frac{2}{7}\right)$$

Which of the following statements is correct ?

(A) Z is minimum at $S\left(\frac{18}{7}, \frac{2}{7}\right)$

(B) Z is maximum at $R\left(\frac{7}{2}, \frac{3}{4}\right)$

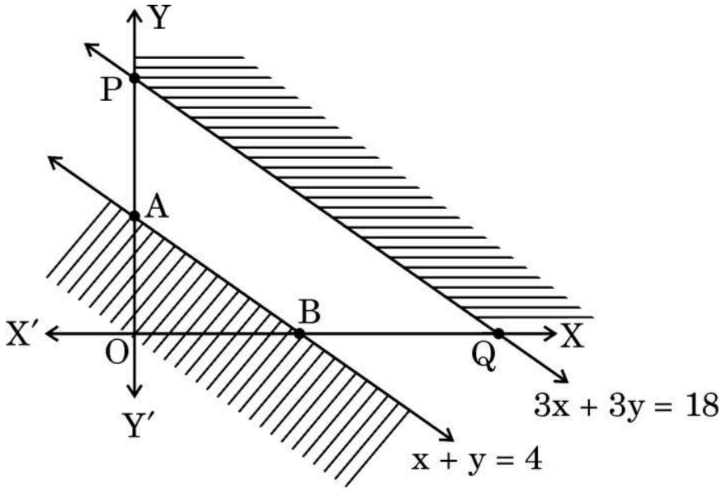
(C) (Value of Z at P) > (Value of Z at Q)

(D) (Value of Z at Q) < (Value of Z at R)

15. In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximised under the following constraints :

$$x + y \le 4,\ \ 3x + 3y \ge 18,\ \ x, y \ge 0$$

Study the graph and select the correct option.



(Note : The figure is not to scale)

The solution of the given LPP :

(A) lies in the shaded unbounded region.

(B) lies in Δ AOB.

(C) does not exist.

(D) lies in the combined region of Δ AOB and unbounded shaded region.

16. Let $|\vec{a}| = 5$ and $-2 \le \lambda \le 1$. Then, the range of $|\lambda \vec{a}|$ is :

(A) [5, 10] (B) [– 2, 5]

(C) [– 2, 1] (D) [–10, 5]

17. The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is :

(A) $\frac{3}{2}$ sq units (B) $\frac{2}{3}$ sq units

(C) 3 sq units (D) $\frac{4}{3}$ sq units

18. A box has 4 green, 8 blue and 3 red pens. A student picks up a pen at random, checks its colour and replaces it in the box. He repeats this process 3 times. The probability that at least one pen picked was red is :

(A) $\frac{124}{125}$ (B) $\frac{1}{125}$

(C) $\frac{61}{125}$ (D) $\frac{64}{125}$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 256$ and $|\vec{b}| = 8$, then $|\vec{a}| = 2$.

Reason (R) : $\sin^2\theta + \cos^2\theta = 1$ and

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$ and $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$.

20. *Assertion (A) :* Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f + g)\, x = e^x + \log x$ where domain of $(f + g)$ is R.

Reason (R) : $\text{Dom}(f + g) = \text{Dom}(f) \cap \text{Dom}(g)$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Find the domain of $f(x) = \sin^{-1}(-x^2)$.

22. (a) Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x > 0$.

OR

(b) If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.

23. Determine the values of x for which $f(x) = \frac{x-4}{x+1}$, $x \neq -1$ is an increasing or a decreasing function.

24. (a) If $\vec{a}$ and $\vec{b}$ are position vectors of point A and point B respectively, find the position vector of point C on BA produced such that BC = 3BA.

OR

(b) Vector $\vec{r}$ is inclined at equal angles to the three axes x, y and z. If magnitude of $\vec{r}$ is $5\sqrt{3}$ units, then find $\vec{r}$.

25. Determine if the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect with each other.

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. Let $A = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -6 & -8 & -4 \end{bmatrix}$ be two matrices. Then, find the matrix B if AB = C.

27. (a) Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x, if $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

OR

(b) Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x, when $x \in (0, 1)$.

28. (a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41$, $x, y \in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not.

OR

(b) Show that the function $f : N \rightarrow N$, where N is a set of natural numbers, given by $f(n) = \begin{cases} n-1, & \text{if } n \text{ is even} \\ n+1, & \text{if } n \text{ is odd} \end{cases}$ is a bijection.

29. Consider the Linear Programming Problem, where the objective function $Z = (x + 4y)$ needs to be minimized subject to constraints

$2x + y \geq 1000$

$x + 2y \geq 800$

$x, y \geq 0.$

Draw a neat graph of the feasible region and find the minimum value of Z.

30. (a) Find the distance of the point P(2, 4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

OR

(b) Let the position vectors of the points A, B and C be $3\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} + 3\hat{k}$ respectively. Find the vector and cartesian equations of the line passing through A and parallel to line BC.

31. A person is Head of two independent selection committees I and II. If the probability of making a wrong selection in committee I is 0·03 and that in committee II is 0·01, then find the probability that the person makes the correct decision of selection :

(i) in both committees

(ii) in only one committee

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Find :

$$\int \frac{x^2+1}{(x-1)^2\,(x+3)}\,dx$$

OR

(b) Evaluate :

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\,dx$$

33. Draw a rough sketch for the curve $y = 2 + |x + 1|$. Using integration, find the area of the region bounded by the curve $y = 2 + |x + 1|$, $x = -4$, $x = 3$ and $y = 0$.

34. (a) Solve the differential equation : $x^2y\,dx - (x^3 + y^3)\,dy = 0$.

OR

(b) Solve the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition $y(0) = 0$.

35. Let the polished side of the mirror be along the line $\frac{x}{1} = \frac{1-y}{-2} = \frac{2z-4}{6}$.

A point P(1, 6, 3), some distance away from the mirror, has its image formed behind the mirror. Find the coordinates of the image point and the distance between the point P and its image.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Three students, Neha, Rani and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads and 2 erasers and pays ₹ 60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹ 90. Sam pays ₹ 70 for 6 pens, 2 notepads and 3 erasers.

Based upon the above information, answer the following questions :

(i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form AX = B. *1*

(ii) Find $|A|$ and confirm if it is possible to find A^{-1}. *1*

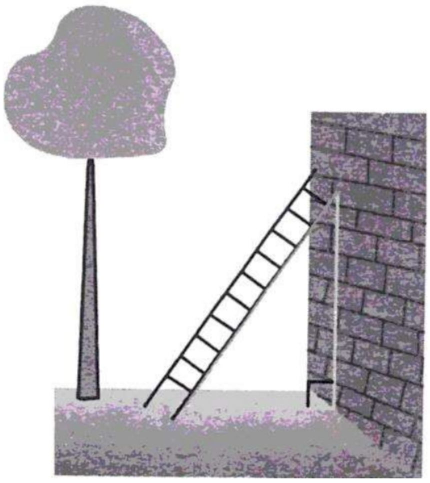
(iii) (a) Find A^{-1}, if possible, and write the formula to find X. *2*

OR

(iii) (b) Find $A^2 - 8I$, where I is an identity matrix. *2*

Case Study – 2

37.



A ladder of fixed length 'h' is to be placed along the wall such that it is free to move along the height of the wall.

Based upon the above information, answer the following questions :

(i) Express the distance (y) between the wall and foot of the ladder in terms of 'h' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer. *1*

(ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point. *1*

(iii) (a) Show that the area (A) of the right triangle is maximum at the critical point. *2*

OR

(iii) (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall ? *2*

Case Study – 3

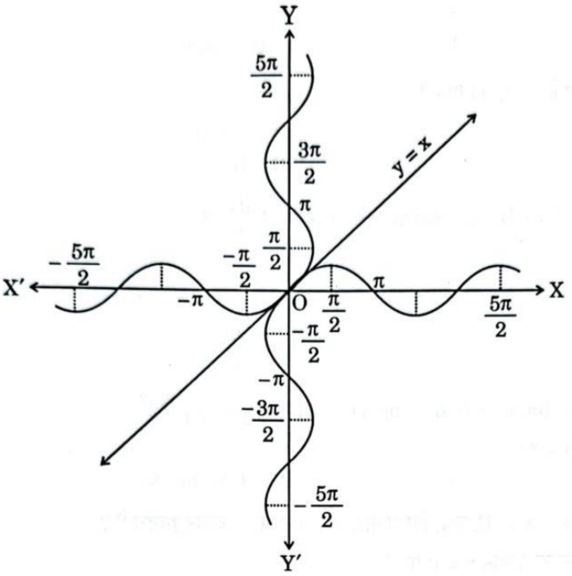
38. A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C.

A person buys a smartphone from this shop.

(i) Find the probability that it was defective. *2*

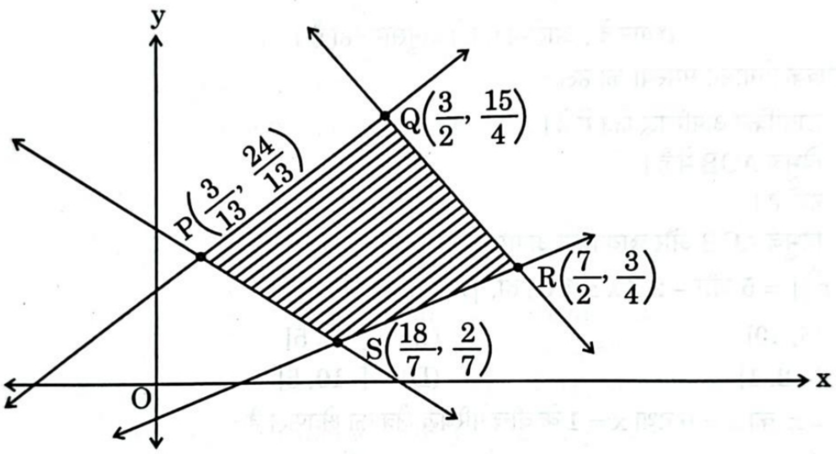
(ii) What is the probability that this defective smartphone was manufactured by company B ? *2*

MARKING SCHEME – 65/6/1

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION-A This section comprises multiple choice questions (MCQs) of 1 mark each.	
1.	**Let both AB′ and B′A be defined for matrices A and B. If order of A is n × m, then the order of B is :** **(A)** n × n **(B)** n × m **(C)** m × m **(D)** m × n	
Ans	(B) n × m	1
2.	**If** $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$**, then A is a/an :** **(A)** scalar matrix **(B)** identity matrix **(C)** symmetric matrix **(D)** skew-symmetric matrix	
Ans	(C) symmetric matrix	1
3.	**The following graph is a combination of :**  **(A)** $y = \sin^{-1} x$ and $y = \cos^{-1} x$ **(B)** $y = \cos^{-1} x$ and $y = \cos x$ **(C)** $y = \sin^{-1} x$ and $y = \sin x$ **(D)** $y = \cos^{-1} x$ and $y = \sin x$	
Ans	(C) $y = \sin^{-1}x$ and $y = \sin x$	1
4.	**Sum of two skew-symmetric matrices of same order is always a/an :** **(A)** skew-symmetric matrix **(B)** symmetric matrix **(C)** null matrix **(D)** identity matrix	

Ans	(A) skew-symmetric matrix	1
5.	$\left[\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ is equal to : (A) $\frac{11\pi}{12}$ (B) $\frac{5\pi}{12}$ (C) $-\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$	
Ans	(D) $\frac{7\pi}{12}$	1
6.	If $f(x) = \begin{cases} \frac{\log(1+ax) + \log(1-bx)}{x}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is : (A) a (B) a + b (C) a – b (D) b	
Ans	(C) a – b	1
7.	If $\tan^{-1}(x^2 - y^2) = a$, where 'a' is a constant, then $\frac{dy}{dx}$ is : (A) $\frac{x}{y}$ (B) $-\frac{x}{y}$ (C) $\frac{a}{x}$ (D) $\frac{a}{y}$	
Ans	(A) $\frac{x}{y}$	1
8.	If $y = a\cos(\log x) + b\sin(\log x)$, then $x^2y_2 + xy_1$ is : (A) cot (log x) (B) y (C) – y (D) tan (log x)	
Ans	(C) –y	1
9.	Let $f(x) = \lvert x \rvert$, $x \in R$. Then, which of the following statements is ***incorrect*** ? (A) f has a minimum value at x = 0. (B) f has no maximum value in R. (C) f is continuous at x = 0. (D) f is differentiable at x = 0.	
Ans	(D) f is differentiable at x = 0	1
10.	Let $f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5$, $f(1) = 0$. Then, f(x) is : (A) $x^3 + 3x^2 + \frac{2}{x^2} + 5x + 11$ (B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$ (C) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$ (D) $x^3 - 3x^2 - \frac{2}{x^2} + 5x - 11$	
Ans	(B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$	1

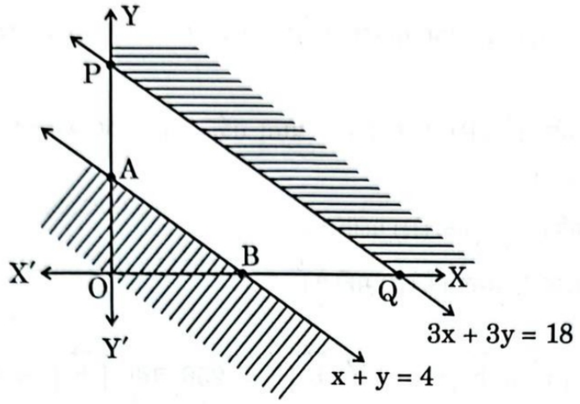
11.	$\int \frac{x+5}{(x+6)^2} e^x \, dx$ is equal to : (A) $\log(x+6) + C$ (B) $e^x + C$ (C) $\frac{e^x}{x+6} + C$ (D) $\frac{-1}{(x+6)^2} + C$	
Ans	(C) $\frac{e^x}{x+6} + C$	1
12.	The order and degree of the following differential equation are, respectively : $-\frac{d^4y}{dx^4} + 2e^{dy/dx} + y^2 = 0$ (A) $-4, 1$ (B) 4, not defined (C) 1, 1 (D) 4, 1	
Ans	(B) 4, not defined	1
13.	The solution for the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is : (A) $3e^{4y} + 4e^{-3x} + C = 0$ (B) $e^{3x+4y} + C = 0$ (C) $3e^{-3y} + 4e^{4x} + 12C = 0$ (D) $3e^{-4y} + 4e^{3x} + 12C = 0$	
Ans	(D) $3e^{-4y} + 4e^{3x} + 12C = 0$	1

14.	For a Linear Programming Problem (LPP), the given objective function is $Z = x + 2y$. The feasible region PQRS determined by the set of constraints is shown as a shaded region in the graph.  (Note : The figure is not to scale) $P \equiv \left(\frac{3}{13}, \frac{24}{13}\right)$, $Q \equiv \left(\frac{3}{2}, \frac{15}{4}\right)$, $R \equiv \left(\frac{7}{2}, \frac{3}{4}\right)$, $S \equiv \left(\frac{18}{7}, \frac{2}{7}\right)$ Which of the following statements is correct ? (A) Z is minimum at $S\left(\frac{18}{7}, \frac{2}{7}\right)$ (B) Z is maximum at $R\left(\frac{7}{2}, \frac{3}{4}\right)$ (C) (Value of Z at P) > (Value of Z at Q) (D) (Value of Z at Q) < (Value of Z at R)	
Ans	(A) Z is minimum at S $\left(\frac{18}{7}, \frac{2}{7}\right)$	1

15. In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximised under the following constraints :

$x + y \leq 4,\ 3x + 3y \geq 18,\ x, y \geq 0$

Study the graph and select the correct option.



(Note : The figure is not to scale)

The solution of the given LPP :

(A) lies in the shaded unbounded region.

(B) lies in Δ AOB.

(C) does not exist.

(D) lies in the combined region of Δ AOB and unbounded shaded region.

Ans (C) does not exist — 1

16. Let $|\vec{a}| = 5$ and $-2 \leq \lambda \leq 1$. Then, the range of $|\lambda \vec{a}|$ is :

(A) [5, 10] (B) [– 2, 5]

(C) [– 2, 1] (D) [–10, 5]

Ans 1 mark for any attempt as correct answer is not given in any option — 1

17. The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is :

(A) $\frac{3}{2}$ sq units (B) $\frac{2}{3}$ sq units

(C) 3 sq units (D) $\frac{4}{3}$ sq units

Ans (D) $\frac{4}{3}$ sq units — 1

18. A box has 4 green, 8 blue and 3 red pens. A student picks up a pen at random, checks its colour and replaces it in the box. He repeats this process 3 times. The probability that at least one pen picked was red is :

(A) $\frac{124}{125}$ (B) $\frac{1}{125}$

(C) $\frac{61}{125}$ (D) $\frac{64}{125}$

Ans (C) $\frac{61}{125}$ — 1

	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* If $\|\vec{a} \times \vec{b}\|^2 + \|\vec{a} \cdot \vec{b}\|^2 = 256$ and $\|\vec{b}\| = 8$, then $\|\vec{a}\| = 2$. *Reason (R) :* $\sin^2 \theta + \cos^2 \theta = 1$ and $\|\vec{a} \times \vec{b}\| = \|\vec{a}\|\|\vec{b}\| \sin \theta$ and $\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\| \cos \theta$.	
Ans	(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).	1
20.	*Assertion (A) :* Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f + g)\, x = e^x + \log x$ where domain of $(f + g)$ is R. *Reason (R) :* $\text{Dom}(f + g) = \text{Dom}(f) \cap \text{Dom}(g)$.	
Ans	(D) Assertion (A) is false, but Reason (R) is true.	1
	SECTION-B This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.	
21.	Find the domain of $f(x) = \sin^{-1}(-x^2)$.	
Ans	(a) $-1 \le -x^2 \le 1 \Rightarrow -1 \le -x^2 \le 0$ $\Rightarrow 0 \le x^2 \le 1 \Rightarrow -1 \le x \le 1$	1 1
22.	(a) Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x > 0$. **OR** (b) If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.	
Ans	(a) Let u= $\sqrt{e^{\sqrt{2x}}}$ and $v = e^{\sqrt{2x}}$ Derivative of $\sqrt{v}$ w.r.t. $v = \frac{1}{2\sqrt{v}}$.	½ 1

	Required derivative = $\frac{1}{2\sqrt{e^{\sqrt{2x}}}}$. OR Taking log on both sides, we get y logx = x logy Differentiating both sides w.r.t. x, we get $\frac{y}{x.} + \log x \frac{dy}{dx} = \frac{x}{y.}\frac{dy}{dx} + \log y$ $\Rightarrow \frac{dy}{dx} = \frac{y(x\log y - y)}{x(y\log x - x)}$	½ ½ 1 ½
23.	**Determine the values of x for which $f(x) = \frac{x-4}{x+1}$, $x \neq -1$ is an increasing or a decreasing function.**	
Ans	$f'(x) = \frac{x+1-x+4}{(x+1)^2} = \frac{5}{(x+1)^2} > 0$ Hence f is increasing in its domain.	$1\frac{1}{2}$ ½
24.	**(a) If $\vec{a}$ and $\vec{b}$ are position vectors of point A and point B respectively, find the position vector of point C on BA produced such that BC = 3BA.** **OR** **(b) Vector $\vec{r}$ is inclined at equal angles to the three axes x, y and z. If magnitude of $\vec{r}$ is $5\sqrt{3}$ units, then find $\vec{r}$.**	
Ans	(a) C divides BA in the ratio 3 : 2 externally Required vector = $\vec{c} = \frac{3\vec{a}-2\vec{b}}{3-2} = 3\vec{a} - 2\vec{b}$  OR (b) Unit vector equally inclined along coordinate axes is $\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$ $\vec{r} = 5\sqrt{3}(\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}) = 5\hat{i} + 5\hat{j} + 5\hat{k}$ (or $-5\hat{i} - 5\hat{j} - 5\hat{k}$)	1 1 1 1
25.	**Determine if the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect with each other.**	
Ans	$\vec{b}_1 = 3\hat{i} - \hat{j}$, $\vec{b}_2 = 2\hat{i} + 3\hat{k}$, $\vec{a}_2 = 4\hat{i} - \hat{k}$, $\vec{a}_1 = \hat{i} + \hat{j} - \hat{k}$ $(\vec{a}_2 - \vec{a}_1) = 3\hat{i} - \hat{j}$ $\vec{b}_1 \times \vec{b}_2 = -3\hat{i} - 9\hat{j} + 2\hat{k}$	 ½ 1

	$(\vec{a_2} - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = (3\hat{i} - \hat{j}).(-3\hat{i} - 9\hat{j} + 2\hat{k}) = 0$, hence lines intersect	½
	SECTION-C This section comprises 6 Short Answer (SA) type questions of 3 marks each.	
26.	***is section comprises 6 Short Answer (SA) type questions of 3 marks each.*** **Let $A = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -6 & -8 & -4 \end{bmatrix}$ be two matrices. Then, find the matrix B if AB = C.**	
Ans	(a) Let B = [x y z] $AB = C \Rightarrow \begin{bmatrix} x & y & z \\ 4x & 4y & 4z \\ -2x & -2y & -2z \end{bmatrix} = \begin{bmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -6 & -8 & -4 \end{bmatrix}$ which gives x = 3, y = 4 and z = 2 B = [3 4 2]	½ 2 ½
27.	**27. (a) Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x, if $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.** **OR** **(b) Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x, when $x \in (0, 1)$.**	
Ans	(a) x = sint gives $y = \sin^{-1}(\sin 3t) = 3t = 3\sin^{-1}x$ $\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$ Aliter: $\frac{dy}{dx} = \frac{3-12x^2}{\sqrt{1-(3x-4x^3)^2}}$ OR (b) x = tant gives $y = \cos^{-1}(\cos 2t) = 2t = 2\tan^{-1}x$ $\frac{dy}{dx} = \frac{2}{1+x^2}$ Aliter: $\frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{-4x}{(1+x^2)^2}$	$\frac{1}{2}+1+\frac{1}{2}$ 1 3 $\frac{1}{2}+1+\frac{1}{2}$ 1 3

28.	(a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41$, $x, y \in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not. **OR** (b) Show that the function $f : N \to N$, where N is a set of natural numbers, given by $f(n) = \begin{cases} n-1, \text{ if n is even} \\ n+1, \text{ if n is odd} \end{cases}$ is a bijection.	
Ans	(a) R = {(1,39), (2, 37),_ _ _ ,(20, 1)} Domain = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20} Range ={1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39} (1, 1) does not belong to R hence not reflexive (1, 39) belongs to R but (39, 1) does not belong to R hence not symmetric (11, 19) and (19, 3) belong to R but (11, 3) does not belong to R hence not transitive. Hence R is not an equivalence relation. OR (a) Let f(x) = f(y) Let x and y are both odd or both even Then either x+1 = y + 1 or x-1 = y-1 gives x = y x odd and y even is rejected as x + 1 = y -1 gives x – y = -2 not possible as odd number and even number cannot differ by 2 Hence f is one-one For onto: Let f(x) = y gives x = y + 1 or x = y - 1 If y is odd, x is even and if y is even, x is odd. Range = N = co-domain, hence onto As f is both one-one and onto hence bijective	 ½ ½ ½ ½ 1 ½ ½ ½ 1 ½
29.	**Consider the Linear Programming Problem, where the objective function Z = (x + 4y) needs to be minimized subject to constraints** $2x + y \geq 1000$ $x + 2y \geq 800$ $x, y \geq 0$. **Draw a neat graph of the feasible region and find the minimum value of Z.**	

Ans

Correct Graph and shading: — 1 ½

Corner points	Value of Z
(800, 0)	800
(400, 200)	1200
(0, 1000)	4000

— 1

$x + 4y < 800$ has no region common with feasible region, hence 800 is minimum — ½

30.

(a) Find the distance of the point P(2, 4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

OR

(b) Let the position vectors of the points A, B and C be $3\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} + 3\hat{k}$ respectively. Find the vector and cartesian equations of the line passing through A and parallel to line BC.

Ans

(a) Let $\vec{a}_2 = 2\hat{i} + 4\hat{j} - \hat{k}$, $\vec{a}_1 = -5\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$

Distance between point and line is given by $d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$

Here $(\vec{a}_2 - \vec{a}_1) = 7\hat{i} + 7\hat{j} - 7\hat{k}$ — ½

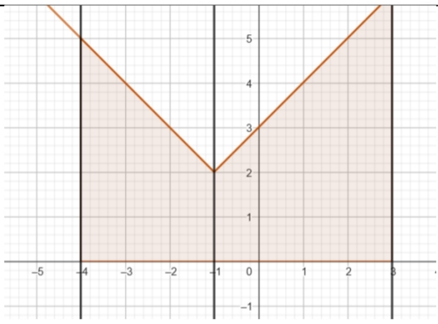
$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = -35\hat{i} + 56\hat{j} + 21\hat{k}$ — $1\frac{1}{2}$

$d = \frac{49\sqrt{2}}{7\sqrt{2}} = 7$ — 1

OR

(b) Vector parallel to given line $= 3\hat{j} + 4\hat{k}$ — 1

	Vector equation is $\vec{r} = 3\hat{i} - \hat{j} - 2\hat{k} + \mu(3\hat{j} + 4\hat{k})$ Cartesian equation is $\frac{x-3}{0} = \frac{y+1}{3} = \frac{z+2}{4}$	1 1
31.	**31.** **A person is Head of two independent selection committees I and II. If the probability of making a wrong selection in committee I is 0·03 and that in committee II is 0·01, then find the probability that the person makes the correct decision of selection :** **(i) in both committees** **(ii) in only one committee**	
Ans	(i) P(correct decision in both committees) =(1- 0.03) . (1- 0.01) = 0.9603 (ii) P(correct decision in one committee) = 0.03 . (1- 0.01) + (1- 0.03). 0.01 = 0.0394	$1 + \frac{1}{2}$ 1 $\frac{1}{2}$
	SECTION-D This section comprises 4 Long Answer (LA) type questions of 5 marks each.	
32.	**(a) Find :** $\int \frac{x^2+1}{(x-1)^2(x+3)} dx$ **OR** **(b) Evaluate :** $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$	
Ans	(a) $\frac{x^2+1}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3} = \frac{3/8}{x-1} + \frac{1/2}{(x-1)^2} + \frac{5/8}{x+3}$ $I = \frac{3}{8}\log\|x-1\| - \frac{1}{2(x-1)} + \frac{5}{8}\log\|x+3\| + C$ OR (b) Let $I = \int_0^{\frac{\pi}{2}} \frac{x}{\sin x+\cos x} dx$ $I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}-x}{\cos x+\sin x} dx$ using property $2I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}}{\sin x+\cos x} dx$ $= \frac{\pi}{2}\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{2}} \frac{1}{\sin(\frac{\pi}{4}+x)} dx$ $2I = \frac{\pi}{2\sqrt{2}}\log\left\|\operatorname{cosec}\left(\frac{\pi}{4}+x\right) - \cot\left(\frac{\pi}{4}+x\right)\right\|_0^{\frac{\pi}{2}}$	1 + 2 2 1 1 1 1

	$I = \frac{\pi}{4\sqrt{2}} log \frac{\sqrt{2}+1}{\sqrt{2}-1}$	1
33.	**Draw a rough sketch for the curve y = 2 + |x + 1|. Using integration, find the area of the region bounded by the curve y = 2 + |x + 1|, x = – 4, x = 3 and y = 0.**	
Ans	 Correct Graph: Required area = $\int_{-4}^{-1}(2 + \|x + 1\|)\,dx + \int_{-1}^{3}(2 + \|x + 1\|)\,dx$ $= \int_{-4}^{-1}(1 - x)\,dx + \int_{-1}^{3}(3 + x)\,dx$ $= -\frac{(1-x)^2}{2}\Big\|_{-4}^{-1} + \frac{(3+x)^2}{2}\Big\|_{-1}^{3}$ $= \frac{21}{2} + 16 = \frac{53}{2}$	1 1 1 1 1
34.	**34. (a) Solve the differential equation : $x^2y\,dx - (x^3 + y^3)\,dy = 0$.** **OR** **(b) Solve the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition y(0) = 0.**	
Ans	(a) Given differential equation can be written as $\frac{dy}{dx} = \frac{yx^2}{x^3+y^3}$ Put y = vx, so $\frac{dv}{dx} = v + x\frac{dv}{dx}$ Therefore, $v + x\frac{dv}{dx} = \frac{vx^3}{x^3+v^3x^3} = \frac{v}{1+v^3}$ $x\frac{dv}{dx} = \frac{-v^4}{1+v^3}$ $(\frac{1}{v^4} + \frac{1}{v})dv = \frac{-dx}{x}$ Integrating we get $\frac{-1}{3v^3} + \log\|v\| = -\log\|x\| + C$	½ 1 1 1 1

	$\frac{-x^3}{3y^3} + \log\lvert y\rvert = C$	½
	OR	
	(b) Given D.E. is $\frac{dy}{dx} + \frac{2x}{1+x^2}\, y = \frac{4x^2}{1+x^2}$	1
	Integrating factor is $e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$	1
	Solution is $y(1+x^2) = \int 4x^2 dx + C$	1
	$y(1+x^2) = \frac{4x^3}{3} + C$	1
	$y(0) = 0$ gives $C = 0$, hence solution is $y(1+x^2) = \frac{4x^3}{3}$	1
35.	**Let the polished side of the mirror be along the line $\frac{x}{1} = \frac{1-y}{-2} = \frac{2z-4}{6}$. A point P(1, 6, 3), some distance away from the mirror, has its image formed behind the mirror. Find the coordinates of the image point and the distance between the point P and its image.**	
Ans	Equation of given line is $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$	1
	Let coordinates of point on the line be $(\lambda, 2\lambda+1, 3\lambda+2)$ for some λ	½
	Drs of line perpendicular to line along mirror are $<\lambda-1, 2\lambda-5, 3\lambda-1>$	½
	$(\lambda-1).1 + (2\lambda-5).2 + (3\lambda-1).3 = 0$ gives $\lambda = 1$	1
	Coordinates of foot of perpendicular are (1,3,5)	1
	For image	
	$\frac{x+1}{2} = 1, \frac{y+6}{2} = 3, \frac{z+3}{2} = 5$ gives image as (1, 0, 7)	½
	Required distance = $\sqrt{0+36+16} = 2\sqrt{13}$	½
	SECTION-E This section comprises 3 case study-based questions of 4 marks each	
36.	**Case Study – 1** **Three students, Neha, Rani and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads and 2 erasers and pays ₹ 60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹ 90. Sam pays ₹ 70 for 6 pens, 2 notepads and 3 erasers.** **Based upon the above information, answer the following questions :** **(i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form AX = B.** *1* **(ii) Find $\lvert A\rvert$ and confirm if it is possible to find A^{-1}.** *1* **(iii) (a) Find A^{-1}, if possible, and write the formula to find X.** *2* **OR** **(iii) (b) Find $A^2 - 8I$, where I is an identity matrix.** *2*	

<table>
<tr><td>Ans</td><td>(i)Let the price of each pen, notepad, eraser be ₹x, ₹y and ₹z respectively
Given system in the form AX = B is $\begin{pmatrix}4&3&2\\2&4&6\\6&2&3\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}60\\90\\70\end{pmatrix}$

(ii)$|A| = 50 \neq 0$, hence A^{-1}exists

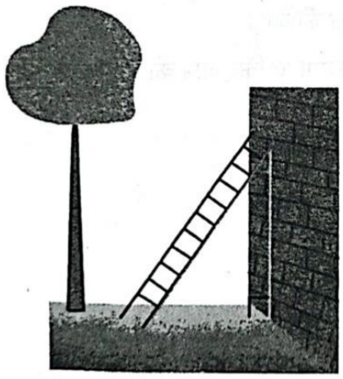
(iii) (a) $A^{-1}=\frac{\text{adj}A}{|A|}=\frac{1}{50}\begin{pmatrix}0&-5&10\\30&0&-20\\-20&10&10\end{pmatrix}$
$X = A^{-1}B$
OR
(iii)(b) $A^2=\begin{pmatrix}4&3&2\\2&4&6\\6&2&3\end{pmatrix}\begin{pmatrix}4&3&2\\2&4&6\\6&2&3\end{pmatrix}=\begin{pmatrix}34&28&32\\52&34&46\\46&32&33\end{pmatrix}$
$A^2 - 8I = \begin{pmatrix}26&28&32\\52&26&46\\46&32&25\end{pmatrix}$</td><td>1

1

1 ½

½

1 ½

½</td></tr>
<tr><td>37.</td><td>Case Study – 2

A ladder of fixed length 'h' is to be placed along the wall such that it is free to move along the height of the wall.
Based upon the above information, answer the following questions :
(i) Express the distance (y) between the wall and foot of the ladder in terms of 'h' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer. 1</td><td></td></tr>
</table>

	(ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point. *1* (iii) (a) Show that the area (A) of the right triangle is maximum at the critical point. *2* **OR** (iii) (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall ? *2*	
Ans	(i) $y^2 = h^2 - x^2$	½
	$A = \frac{1}{2}xy = \frac{1}{2}x\sqrt{h^2 - x^2}$	½
	ii) $\frac{dA}{dx} = \frac{1}{2}\sqrt{h^2 - x^2} + \frac{1}{2}x\frac{-x}{\sqrt{h^2 - x^2}}$	½
	$\frac{dA}{dx} = 0$ gives $x = \frac{h}{\sqrt{2}}$	½
	(iii)(a) $A'' = \frac{1}{2}\frac{-4x.\sqrt{h^2 - x^2} - (h^2 - 2x^2)\frac{-x}{\sqrt{h^2 - x^2}}}{h^2 - x^2}$ is < 0 at $x = \frac{h}{\sqrt{2}}$	$1\frac{1}{2}$
	Hence A is maximum at critical point	½
	OR	
	(iii)(b) $y^2 = 25 - x^2$ hence $y = 3$ gives $x = 4$	½
	$2y\frac{dy}{dt} = -2x\frac{dx}{dt}$	1
	$\frac{dx}{dt} = 1.5 m/s$	½

38.	A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C. A person buys a smartphone from this shop. (i) Find the probability that it was defective. **2** (ii) What is the probability that this defective smartphone was manufactured by company B ? **2**	
Ans	(i) P(defective smartphone) = $0.25\times0.05 + 0.35\times 0.04 + 0.40\times0.02$	1 ½
	$= 0.0345$	½
	(ii) P(B/ Defective) = $\frac{0.35\times 0.04}{0.25\times 0.05 + 0.35\times0.04 + 0.40\times0.02}$	1 ½
	$= \frac{140}{345}$ or $\frac{28}{69}$	½